# Anomalous diffusion of charged particles in a lattice Lorentz gas in a transverse magnetic field

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(Received 9 September 1998)

We study diffusion in the Lorentz gas of noninteracting charged particles on a triangular lattice in a transverse magnetic field. The percolation threshold appears at the scatterers concentration $c_0=0.2155$. The diffusion at $c_0$ is anomalous with the same exponent $d_w=2.874$ as the one found for the universality class of the standard two-dimensional lattice percolation. The presence of logarithmic corrections to the $t^{-2}$ algebraic tail of the velocity autocorrelation function is demonstrated in a special case for concentration of scatterers close to 1, by making use of the moment propagation technique. The results of our computer simulations show that the diffusion coefficient has a maximum above $c_0$, and the velocity autocorrelation function changes the sign of its amplitude, from negative to positive, at intermediate scatterers concentration. We employ the Boltzmann approximation and calculate the diffusion coefficient and the velocity autocorrelation function. [S1063-651X(99)04403-7]

PACS number(s): 05.40.−a, 64.60.Ak 66.10.Cb

## I. INTRODUCTION

Magnetotransport experiments in two-dimensional antidot arrays [1–5] revealed a characteristic magnetoresistivity oscillation at low magnetic fields. In such systems antidot holes form a superlattice with a period of a few hundred nanometers in a two-dimensional electron gas. Hence, the electron motion in antidot arrays can be described using classical dynamics, which is confirmed by numerical calculations [2,6,7]. Experiments performed on disordered antidot arrays [8–10] showed that the peaks in magnetoresistivity vanish in the presence of a short range disorder, i.e., antidots are randomly distorted from their superlattice positions. This effect was confirmed by a computer simulation of the classical model [11].

Bobylev *et al.* [12] discussed the problems arising when one attempts to employ the kinetic theory to investigate two-dimensional magnetotransport in a system with completely disordered stationary scatterers, i.e., in Lorentz gas. Their model consists of noninteracting charged particles moving under the influence of the transverse magnetic field $B\hat{\mathbf{z}}$ in the $(x,y)$ plane with randomly distributed hard disks playing the role of stationary scatterers. It has been shown that in this model the Boltzmann equation is not valid even at low densities of scatterers. In the absence of an electric field the continuum percolation problem arises. This means that electrons moving on cyclotron orbits with a radius smaller than the critical radius are trapped in the finite clusters of scatterers. Thus there is no diffusion below the percolation threshold [12]. An unexpected result was obtained for a small in-plane electric field and perpendicular magnetic field [13]. The electron drifting in the empty space can be trapped by a single scatterer with a short range repulsive interaction.

It is very difficult to obtain highly accurate results in computer simulations performed in the continuum phase space especially if one examines the long-time tails of the velocity autocorrelation function, anomalous diffusion in continuum percolation, etc. In recent years, lattice-gas models have been applied to atomic fluids for testing concepts of the kinetic theory [14–17]. The important result of these investigations was discovering the algebraic long-time tails of the velocity autocorrelation function of a tagged particle. This function decays with the exponent $d/2$ for the fluid and $d/2+1$ for the Lorentz gas, where $d$ is a space dimensionality. Investigation of the long-time correlations by a computer simulation was possible due to the application of the very accurate moment propagation method proposed by van der Hoef and Frenkel [16]. This technique was also applied to calculate the algebraic decay of the velocity autocorrelation in the problem of random walk on percolation clusters [18].

In this paper we propose a lattice version of the Lorentz gas in the transverse magnetic field and use it to study diffusion at and above the percolation threshold. In Sec. II we define a discrete model. The calculation of the percolation threshold is presented in Sec. III. The important quantity in the transport problem is the velocity autocorrelation function, whose long-time tails and amplitude sign change are studied in Sec. IV by making use of the moment propagation technique. In Sec. V we apply the Boltzmann approximation to calculate the diffusion coefficient and the role of correlated collisions is discussed.

## II. THE DISCRETE MODEL

We consider the Lorentz gas on a triangular lattice with a fraction of sites $c$ occupied randomly by stationary scatterers. The noninteracting charged particles travel over one lattice constant per unit time step in the direction of their velocities. The particles have the same speed and their velocity directions are restricted to the following lattice vectors:

$$\mathbf{e}_n=\left(\cos\left(\frac{2\pi n}{6}\right),-\sin\left(\frac{2\pi n}{6}\right)\right),\quad n=0,1,\ldots,5. \tag{1}$$

It is convenient to set the speed of particles as well as the lattice constant both equal to 1. The presence of the magnetic field $B\hat{\mathbf{z}}$ perpendicular to the particle velocities forces charged particles to move along closed orbits. In our discrete model the cyclotron orbit is formed by vertices of the hexa-

 

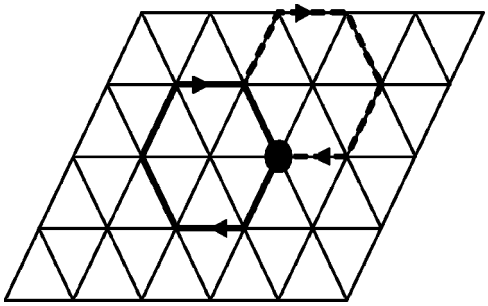

FIG. 1. An example of two orbits (hexagons) on the triangular lattice crossing the site occupied by a scatterer represented by filled circle.

gon whose side is equal to the lattice constant. Hence the magnetic field rotates particles by $2\pi/6$ clockwise in each unit time step. When a particle enters a site occupied by a scatterer, its velocity direction changes according to stochastic rules. Here we assume isotropic scattering rules, i.e., post-collisional velocity directions have equal probabilities ($\frac{1}{6}$). It is worth noting that the scattered particle changes its cyclotron orbit (see Fig. 1). We assume that the cyclotron orbit of a particle should be uniquely determined by its position and velocity. The other possible orbit consistent with this assumption is formed by vertices of a triangle with the side equal to the lattice constant for which the rotation angle is $2\pi/3$. The latter case will not be discussed in this paper. Thus, the concentration of scatterers becomes a parameter in this model and the magnetic field is considered to be constant.

Since our model belongs to the class of probabilistic cellular automata, it is well suited for a computer simulation. It is convenient to define the distribution function $P(n,\mathbf{r},t)$—the probability that a particle being at time $t$ at a site $\mathbf{r}$ has a velocity directed along $\mathbf{e}_n$. The evolution of $P(n,\mathbf{r},t)$ can be written in two steps.

(1) *Rotation*, $P(n+1,\mathbf{r},t+1)=P(n,\mathbf{r}-\mathbf{e}_n,t)$, at a site $\mathbf{r}$ without a scatterer.

(2) *Scattering*, $P(n,\mathbf{r},t+1)=\frac{1}{6}\Sigma_{l=0}^{5}P(l,\mathbf{r}-\mathbf{e}_l,t)$, at a site $\mathbf{r}$ occupied by a scatterer.

Knowing the initial value of the distribution function and the positions of scatterers one can perform accurate computer simulations.

## III. THE PERCOLATION THRESHOLD

In order to study the macroscopic transport (diffusion) we have to solve the percolation problem, i.e., to find the critical value $c_0$ of a concentration of scatterers at which the infinite (spanning) cluster occurs. However, the word ''cluster'' has a meaning different than in the classical percolation problem. In our model two scatterers belong to the same cluster if they are connected by a path formed by cyclotron orbits. A neighbor of a given scatterer in the cluster can be placed at one of the 18 sites that belong to 6 orbits crossing the site occupied by the scatterer. To find the percolation threshold $c_0$ we performed the computer simulation on the lattice of $L\times L$ sites and measured the probability $p(c,L)$ that the spanning cluster occurs over the scale $L$ at concentration of random scatterers $c$ (see Fig. 2). We generated $10^4$ independent scatterer configurations for given $c$ and $L$ and then the probability $p(c,L)$ was calculated as a fraction of configurations with the spanning clusters. Using the results obtained for the lattice size $L=125,250,375,500$, and 750, and applying Kirkpatrick's method [19], i.e., extrapolation of the fixed points $c^*(L)$ obtained from the equation $p(c,L)=c$, and extrapolation of points $\tilde{c}(L)$ defined by the condition $p(c,L)=\frac{1}{2}$, we found $c_0=0.2155\pm0.0005$. This value is much lower than the threshold value for the classical site percolation problem on the triangular lattice ($c_0=\frac{1}{2}$) [20], where the random walk over the occupied nearest neighbor sites is considered. Therefore, it is interesting to check whether our model belongs to the universality class of the two-dimensional lattice percolation [21]. To answer this question we calculate the critical exponent $d_w$ of the mean-square displacement $R^2(t)\sim t^{2/d_w}$ at the percolation threshold and above it.

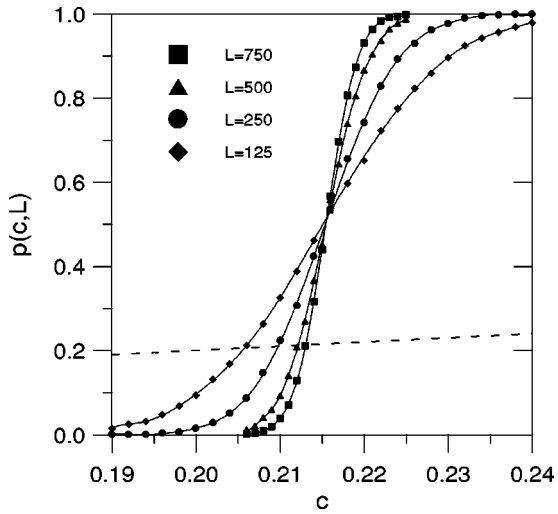


FIG. 2. The probability that a spanning cluster occurs at the concentration of scatterers $c$ on the lattice of $L\times L$ sites for $L=125,250,500$, and 750. The symbols represent Monte Carlo simulation data. The dashed line, $f(c)=c$, is used to find the fixed points.

## IV. DIRECT CALCULATION OF THE VELOCITY AUTOCORRELATION FUNCTION

Similar to an earlier work on the percolation problem [18], we simulated directly the velocity autocorrelation function (VAF) $\Phi_x(t)=\langle v_x(0)v_x(t)\rangle$ using a very accurate moment propagation technique (for details see Ref. [16]). It is worth noting that statistical errors remain two orders of magnitude lower than the VAF even for $10^3$ mean-free time steps. On the other hand, due to large statistical errors, it is practically impossible to calculate VAF for even 100 mean-free time steps by the Monte Carlo method, which is still in use for stochastic Lorentz gases [22].

In order to study the long-time behavior of VAF and the mean-square displacement, we consider a diffusion only on the spanning cluster. First, we have to find the spanning cluster on the triangular lattice with $L\times L$ sites for a given configuration of scatterers. As the initial distribution $P(n,\mathbf{r},0)$ we have to use the steady-state solution of the distribution

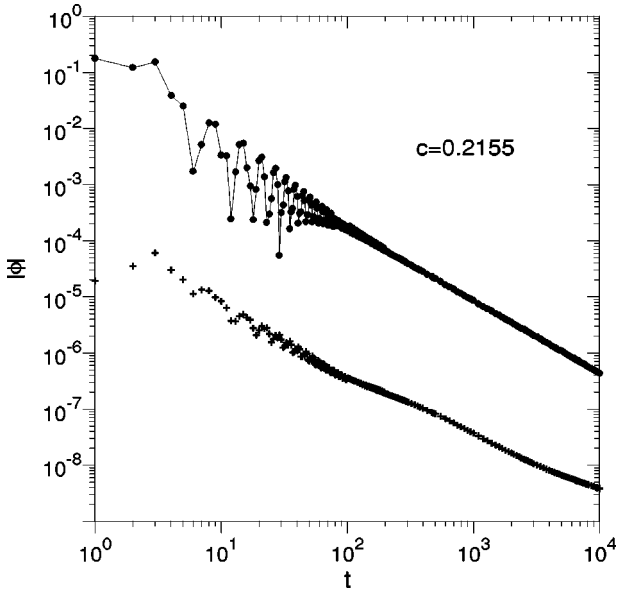


FIG. 3. A log-log plot of the absolute value of the velocity autocorrelation function as a function of time $t$ at the percolation threshold. The simulation data points are denoted by circles. The estimated statistical errors are shown as crosses (lower curve). Dimensionless units.

function $P(n,\mathbf{r},t)$. It has a constant (nonzero) value only for the sites and velocity directions determined by all cyclotron orbits of the spanning cluster. In the moment propagation method, however, we calculate another function $W(n,\mathbf{r},t)$ with the same evolution rules as for $P(n,\mathbf{r},t)$

$$W(n,\mathbf{r},t+1)=\begin{cases} W(n-1,\mathbf{r}-\mathbf{e}_{n-1},t), & \text{for} \quad \theta_{\mathbf{r}}=0 \\ \frac{1}{6}\sum_{l=0}^{5} W(l,\mathbf{r}-\mathbf{e}_l,t), & \text{for} \quad \theta_{\mathbf{r}}=1, \end{cases} \tag{2}$$

with the initial conditions

$$W(n,\mathbf{r},0)=P(n,\mathbf{r},0)e_{nx}.$$

Here $\theta_{\mathbf{r}}=1$ or 0, depending on whether the site $\mathbf{r}$ is or is not occupied by a scatterer. Then, VAF can be calculated from the formula

$$\Phi_x(t)=\left\langle \sum_{r,n} W(n,\mathbf{r},t)e_{nx} \right\rangle, \tag{3}$$

where averaging is over the different spanning clusters. It is worth noting that the moment propagation technique is exact for the single cluster because all particle trajectories are accounted for. The statistical errors occur due to averaging over different clusters (100 in our simulation). The result obtained for the lattice with $L=500$ and for $t\leqslant 10^4$ shows that VAF has a long-time tail (see Fig. 3) given by the algebraic formula $\Phi_x(t)\sim t^{-b}$ with $b=1.305\pm 0.01$ at the percolation threshold and $b=2$ above the threshold. We observed also a short period oscillation superimposed on the decay of VAF that plays an important role for several hundred time steps. They are generated by the motion of the particle on cyclotron orbits.

### A. Change of the sign of VAF

An interesting problem encountered in the simulation data is the sign change of the correlation with the increase of the scatterers concentration. At the percolation threshold and just above it, VAF has a negative value at long times, similar to VAF for the random walk problem on percolation clusters [18,23]. The same sign of the correlation was observed for VAF in ballistic lattice Lorentz gases [17]. However, VAF in our model becomes a positively defined function at intermediate concentrations of scatterers (i.e., for $c>0.52$). The positive correlation was observed in a cellular automata lattice gas and explained by the mode-coupling theory [14].

In order to understand change of the sign of VAF with increasing concentration of scatterers, we study the simplest case where all but one lattice sites are occupied by scatterers. Thus, only one site is unoccupied and it will be called a hole in the sea of scatterers. It is interesting to discuss this case for several reasons.

(i) VAF in this model can be calculated without numerical errors because there is only one configuration of scatterers.

(ii) Nonzero correlations are generated only by the hole. This is implied by the isotropic scattering rules.

(iii) All nonzero contributions to VAF for $t\geqslant 1$ start from six initial positions of the particle only.

(iv) The magnetic field affects the particle trajectories only inside the hole.

Owing to (iv) we can also easily calculate VAF for the system without a magnetic field ($B=0$), in which a particle goes through the hole along a straight line. Property (iii) comes from the fact that the initial distribution $P(n,\mathbf{r},0)=1/(6N)$, for all $\mathbf{r}$ and $n$, where $N$ means a number of the lattice sites ($N=L^2$). Moreover, for a scatterer located at site $\mathbf{r}$ there exists a pair of trajectories beginning at sites $\{\mathbf{r}-\mathbf{e}_n,\mathbf{r}+\mathbf{e}_n\}$ with opposite initial velocities $\{\mathbf{e}_n,-\mathbf{e}_n\}$. These trajectories meet at site $\mathbf{r}$ after first time step and their contribution to VAF cancel each other for $t>1$. Thus, only trajectories starting from the nearest neighbors of the hole with velocities directed towards the hole can contribute to a nonzero correlation. It is worth noting that point (ii) implies that the velocity autocorrelation function is determined only by the return probabilities of the particle to the hole.

Using the moment propagation technique, we calculated VAF on the lattice with $L=500$ with and without magnetic field (see Fig. 4). VAF is negative when a magnetic field is switched off, which means that on average the particle returns to the hole with the velocity opposite to its initial velocity. On the other hand, VAF becomes positive in the presence of a magnetic field. This can be roughly explained as follows. The particle starting with velocity $\mathbf{e_n}$ enters the hole at $t=1$, and its velocity is rotated by the angle of $\pi/3$. Then it enters the sea of scatterers and leaves it (on average) with velocity rotated by $\pi$, as in the case without a magnetic field. Upon returning to the hole the particle velocity is rotated again by $\pi/3$. Thus, the particle velocity is rotated on average by $5\pi/3$ with respect to the initial direction, which implies positive correlations. We can expect that positive correlation occurs also for a hole in a finite ''island'' of scatterers and for systems with many holes.

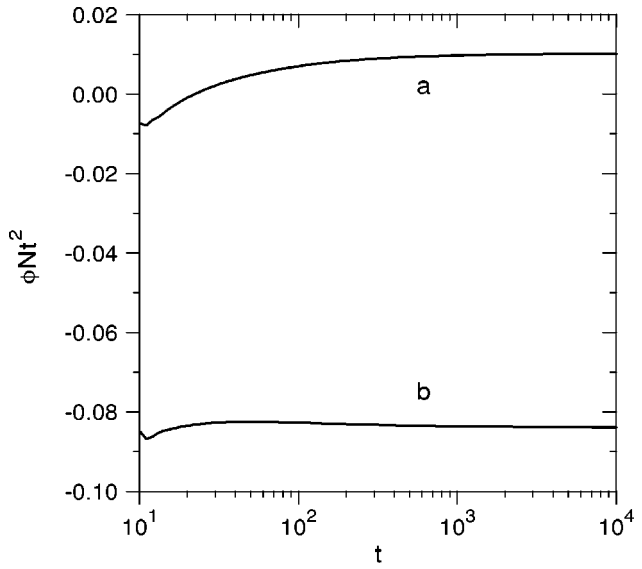


FIG. 4. A semilog plot of $\Phi(t)Nt^2$ as a function of time $t$ for the single hole case. The exact results obtained by the moment propagation technique on the lattice with linear size $L=500$. Curve $a$ ($b$) corresponds to the model with (without) a magnetic field, respectively. Dimensionless units.

Now we consider VAF at the percolation threshold $c=0.2155$ where its sign is negative. The negative correlations are due to the so-called cage effect [24], which can be explained by repeated backscattering of the particle. Positive correlations were observed in the simulations carried out for a concentration $c>0.52$, i.e., above the percolation threshold of the random walk problem. For such concentrations we can distinguish ''islands'' formed by scatterers containing holes. A mean size of an island will increase with concentration, which implies that the effect of positive correlations described above for the single hole will become dominant at high $c$.

### B. Logarithmic correction to the algebraic tail of VAF

In the case of the single hole in the sea of scatterers it is easy to study the influence of the periodic boundary conditions on the result (finite size effect), because in this case VAF can be calculated without statistical errors. Comparing results obtained for different $L$ we found limiting time step $t_{max}$ for each $L$ such that for $t<t_{max}$ VAF does not change with an increase of $L$, e.g., when a magnetic field is included $t_{max}=647,2630,9876$ for $L=125,250,500$, respectively. It is easy to see in Fig. 4 that the $t^{-2}$ tail of VAF is not sufficient to fit the numerical data even for large $t$. Hence, there should be another asymptotic correction to the algebraic tail. The data obtained for both models (with and without magnetic field) and for the linear lattice size $L=500$ are very well fitted by the formula with a logarithmic correction to the $t^{-2}$ decay

$$\Phi(t)=N^{-1}t^{-2}[A_0+A_1t^{-1}\ln(t/A_2)]. \tag{4}$$

The values of the coefficients $A_k$ presented in Table I were obtained from fitting to data of $\Phi(t)Nt^2$ for $t>6000$. It is worth noting that Eq. (4) agrees qualitatively with the kinetic-theory calculations [24] for the site-percolation model. We are able to demonstrate the logarithmic correction to the algebraic tail due to the absence of statistical errors in calculations of VAF for the single hole case. We think that such a correction occurs for a smaller concentration of scatterers too, but then the calculation of VAF requires averaging over different scatterer configurations, which produces statistical errors. Therefore, finding the logarithmic correction in a general case is more difficult in comparison with the single hole case.

TABLE I. Coefficients $A_k$ ($k=0,1,2$) in Eq. (4) obtained by fitting the simulation data of the velocity autocorrelation function in the single hole case with or without magnetic field.

| Magnetic field | $A_0$ | $A_1$ | $A_2$ |
|---|---|---|---|
| $B=0$ | $-0.083863$ | 0.072 | 19.7 |
| $B\neq 0$ | 0.010296 | $-0.078$ | 1.5 |

## V. DIFFUSION

Using the result obtained in the simulation of VAF we have calculated a mean-square displacement along the $x$ direction from the following formula [18]:

$$X^2(t)=t\left[\Phi_x(0)+2\sum_{s=1}^{t-1}\Phi_x(s)\right]-2\sum_{s=1}^{t-1}s\Phi_x(s). \tag{5}$$

Fitting the values of $X^2(t)$ calculated at the percolation threshold, $c_0=0.2155$, to the power law $X^2(t)\sim t^{2/d_w(t)}$, we estimated the exponent $d_w=2.874\pm 0.01$. A value of $d_w>2$ indicates the anomalous diffusion, i.e., the mean-square displacement grows slower than linearly with time. This result agrees very well with the results obtained by the exact enumeration method [25], $d_w=2.86\pm 0.02$, and the moment propagation method [18], $d_w=2.873\pm 0.012$, for the random walk problem on the percolation cluster on a square lattice. This agreement can be easily explained because our model is in some sense also a random walk model. We can describe the diffusion (mean-square displacement) at long times in another way. Instead of following the motion of a particle on cyclotron orbits we can consider the motion of the center of its cyclotron orbit. When the particle moves along a single orbit then the center is at rest. A change of a cyclotron orbit as the result of the particle collision with a scatterer causes a jump of the center of the orbit (see for example Fig. 1). The jumps can have different lengths. Therefore, the motion of the center of the particle's cyclotron orbit is a kind of the random walk with different length jumps and with different waiting times both depending on the local configuration of scatterers.

Above the percolation threshold, we found that the exponent $d_w=2$, which indicates the normal diffusion. Hence, the diffusion coefficient above the threshold can be obtained by taking the limit

$$D=\lim_{t\to\infty}\frac{\langle X^2(t)\rangle}{2t}.$$

We found that $D\neq 0$ above the percolation threshold, reaches its maximum $0.31172\pm 0.00005$ at $c=0.693$ (see

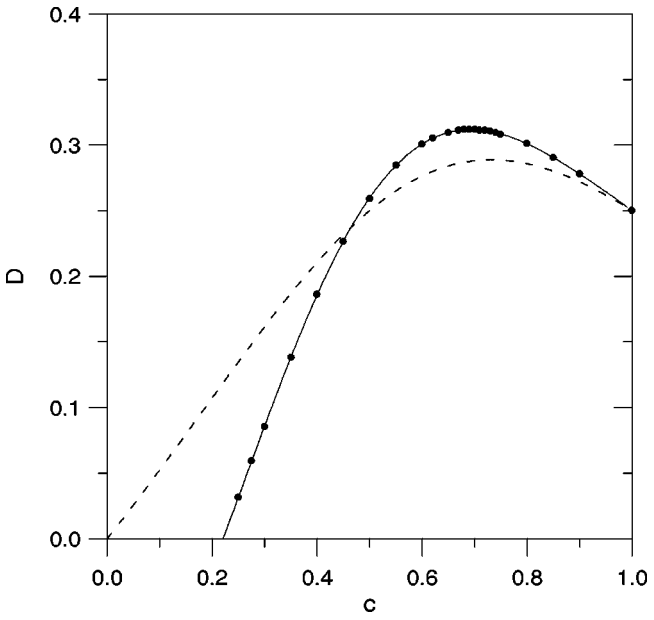


FIG. 5. The diffusion coefficient as a function of the concentration of scatterers. Circles represent the results of computer simulations, and the dashed line shows the results obtained using the Boltzmann approximation. The error bars are smaller than symbol size. Dimensionless units.

Fig. 5), and then decreases. The value $D=\frac{1}{4}$ at $c=1$ is obvious as this case corresponds to the random walk on the regular lattice.

### A. The Boltzmann approximation

It is interesting to find the influence of correlated collisions on the diffusion. In order to answer this question we construct the simplest theoretical approach, the Boltzmann approximation, which assumes a constant frequency of collisions and neglects any correlations between collisions of the particle with scatterers. Hence, we can neglect the spatial dependence of the distribution function $P(n,\mathbf{r},t)$. Furthermore, the probability of changing the particle velocity as the result of scattering is $c$, whereas the probability of its rotation by a magnetic field is $1-c$. Thus the evolution of the six-component vector distribution function $P^B(t)$ can be described by the equation

$$P^B(t+1)=[(1-c)\mathbf{I}+c\mathbf{T}]\mathbf{R}P^B(t), \tag{6}$$

where $\mathbf{R}$ is a rotation matrix with elements $R_{kl}=\Delta(l,k+1)$ defined by the Kronecker delta. The scattering matrix $\mathbf{T}$ for the isotropic scattering rules has all elements equal to $\frac{1}{6}$. The solution of Eq. (6), $P^B(t)$ as a function of $P^B(0)$, can be easily obtained via diagonalization of the transfer matrix. The result can be written in the form

$$P^B(t)=\mathbf{S}\Pi^t\mathbf{S}^\dagger P^B(0), \tag{7}$$

where $\mathbf{\Pi}$ is the diagonal matrix

$$\Pi_{nn}=\begin{bmatrix}1\\(1-c)z_3\\(1-c)z_2\\(1-c)z_4\\(1-c)z_1\\(1-c)z_5\end{bmatrix},\quad \mathbf{S}=\frac{\sqrt{6}}{6}\begin{bmatrix}1&z_3&z_4&z_2&z_5&z_1\\1&1&z_2&z_4&z_4&z_2\\1&z_3&1&1&z_3&z_3\\1&1&z_4&z_2&z_2&z_4\\1&z_3&z_2&z_4&z_1&z_5\\1&1&1&1&1&1\end{bmatrix}, \tag{8}$$

where

$$z_n=e^{i2\pi n/6},$$

and $\mathbf{S}^\dagger$ means the Hermitian conjugation of the matrix $\mathbf{S}$.

VAF calculated in this approach becomes

$$\Phi_x^B(t)=\frac{1}{2}\cos\left(\frac{2\pi t}{6}\right)(1-c)^t. \tag{9}$$

It is the superposition of an exponential decay and the oscillation with period six, which is in disagreement with computer simulation data. The dependence of the diffusion coefficient on the concentration $c$ is expressed by the formula

$$D^B(c)=\frac{c(2-c)}{4(c^2-c+1)}. \tag{10}$$

It has the maximum equal to $\sqrt{3}/6$ at $c=\sqrt{3}-1$. Hence the Boltzmann approximation reduces the maximum and moves its position to a higher concentration, and of course it gives a wrong result below the percolation threshold (see Fig. 5). Note that at $c=0.4056$ the Boltzmann approximation predicts the correct value of the diffusion coefficient, whereas gives wrong values of VAF. This is due to the fact that the VAF amplitude sign changes from negative to positive at intermediate concentration. It is known that the deviation of the computer simulation data of the diffusion coefficient from the Boltzmann prediction is negative for negative VAF [15] and positive for the positive velocity correlation, e.g., in fluids. Positive deviations were observed also in a deterministic lattice Lorentz gas model [26].

In our model the diffusion coefficient is proportional to the conductivity, thus the conductivity vanishes below the percolation threshold. It is an interesting problem to determine the influence of a small electric field on the transport properties near the percolation threshold.

## VI. DISCUSSION

The results of our simulations show that our simple lattice model has all the essential features of the continuous one [12], e.g., it is the percolation model where the Boltzmann approximation fails. Therefore, it would be interesting to check, in the continuous model, our predictions concerning algebraic long-time tails of the autocorrelation functions, a change of sign of correlations and nonmonotonic dependence of the diffusion coefficient on the concentration of scatterers. On the other hand, the result presented here can be used to test numerically a theoretical approach more sophisticated than the one presented here (the Boltzmann approximation). Several different kinetic approximations accounting for cor-

related collisions were discussed by van Velzen [27]. He calculated the diffusion coefficient for a stochastic lattice Lorentz gas on the square lattice using the ring approximation, the repeated ring approximation, the self-consistent ring approximation, the self-consistent repeated ring approximation, and the effective medium approximation. In the ring or repeated ring approximation one takes into account all events with single or multiple returns of the particle to the same scatterer. In the effective medium approximation van Velzen replaced the scattering operator by the effective operator that accounts in a self-consistent way for all repeated ring collisions. Comparison of the diffusion coefficent calculated in different kinetic approximations with the results of computer simulations revealed that only the results obtained in the effective medium approximation agreed with simulation data in the entire interval of scatterers densities. The other approximations break down at low or intermediate densities of scatterers. In the models studied by van Velzen [27] the particle moves along a straight line between successive collisions, therefore his theoretical results cannot be directly applied to our model. It would be interesting to check which kinetic approximations proposed by van Velzen for a stochastic lattice Lorentz gas could correctly describe the properties of the present model, e.g., the percolation threshold and the change of sign of the autocorrelation function.

The model introduced in this paper can be easily extended to different stochastic scattering rules as well as for different lattices. Moreover, it seems that a choice of the deterministic scattering rules (some mirrors can play the role of scatterers) would lead to an interesting percolation model, because in the Lorentz lattice gas with strictly deterministic scattering rules, four classes of diffusive behavior of particles were observed [28], e.g., abnormal diffusion where the distribution function is non-Gaussian, and the mean-square displacement $R^2(t) \sim t^{1-\alpha}$, with $0 < \alpha < 1$.

In this paper we studied the model without the electric field in order to answer some questions concerning the percolation problem. However, it is possible to generalize the discrete model to discuss the role of magnetic and electric fields in magnetotransport, especially the influence of an electric field on the conductivity near the percolation threshold. Switching on an electric field allows one to study magnetotransport in a regular array of scatterers.

Fabrication of antidot arrays reached such a level that now it is possible to prepare complicated patterns, e.g., Penrose lattice [4] or short range disordered arrays [8,9]. To our knowledge there has not yet been performed a magnetotransport experiment in a completely random antidot array. It would be interesting to prepare such arrays and investigate the percolation problem experimentally.


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